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Hovering Motion of a Friction-Driven Gyroscopic Mass

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Nomenclature

A = minor moment of inertia (about axis perpendicular to hole)
M = applied torque due to friction
λ = expression of inertial imbalance; $\lambda+1$ = major moment of inertia divided by minor moment of inertia
ϕ,θ,ψ = Euler angles (see Fig. 1)
ω_c = angular velocity of cavity
ω_r = angular velocity of ball relative to cavity

Reference 1 is a theoretical and experimental treatment of the motion which occurs when a ball is mounted within a spinning spherical cavity. The configuration is one in which a gyroscopic mass (the ball) is driven by means of friction between itself and the spinning cavity. The ball in this case is endowed with a major and minor axis of inertia by the simple means of a hole bored symmetrically through its center. The gyroscopic motion is such that the ball eventually aligns its major axis of inertia (axis through the hole) with the axis of spin of the spherical cavity. A specific application of this phenomenon is described in Ref. 2, where the ball is used as an automatically opening plug in a spinning tubular projectile. In both of these previous works it has been noted that under certain conditions the ball will "hover" about its initial orientation with the result that the alignment of the hole axis with the cavity spin axis is significantly delayed. In this Note, the equations of motion are shown to predict the existence of the hovering mode and this leads to a simple guideline by which the hovering behavior (which is normally undesirable) may be avoided.

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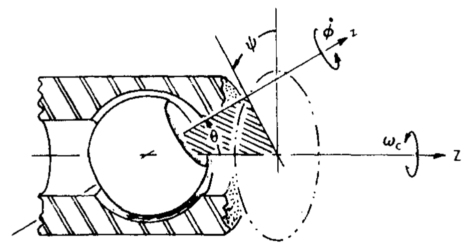

Fig. 1 Sketch defining components of ball motion within the spinning cavity.

Figure 1 illustrates the coordinate systems used in the analysis. The equation governing the angle of tilt (nutation angle) of the ball axis relative to the cavity spin axis is[1]

$$\ddot{\theta} = -(M/A\omega_r)\dot{\theta} - \dot{\psi}\sin\theta[(\lambda+1)\dot{\phi} + \lambda\dot{\psi}\cos\theta] \tag{1}$$

The initial conditions governing the motion are

$$\theta(0)=\theta_0 \qquad \dot{\theta}(0)=0 \qquad \dot{\psi}(0)=\omega_c \qquad \dot{\phi}(0)=0$$

and in the initial instants of motion Eq. (1) may be approximately written as

$$\ddot{\theta}_0 = -(M/A\omega_r)\dot{\theta}_0 - (\lambda/2)\omega_c^2\sin 2\theta_0$$

Here we have retained the term involving $\dot{\theta}_0$ because even though $\dot{\theta}_0$ itself is initially at zero, a large applied moment M may give significance to this term. In addition, in the initial instants of motion, the relative angular velocity ω_r is essentially due to this nutation rate so that $\dot{\theta}_0 \simeq -\omega_r$ and the initial acceleration is given by

$$\ddot{\theta}_0 = M/A + \omega_c^2 D$$

where, as in Ref. 1, $D = (\lambda/2)\sin 2\theta_0$. In situations where hovering occurs, this initial angular acceleration is approximately zero so that if $M \simeq -A\omega_c^2 D$ the condition for hovering is met. For smaller values of M the angular acceleration will be negative and non-zero and the hovering behavior may thereby be avoided by design. A general design guideline may be formulated as follows:

$$H \equiv (M/A\omega_c^2|D|) \leq 1 \tag{2}$$

for the avoidance of hovering.

Further explicit formulation of the value of H requires the expression of the torque term M in terms of the design parameters. Such expressions are given in Refs. 1 and 2. It may be noted, however, that from the definition of D hovering is unavoidable when $\lambda \to 0$ or when $\theta_0 \to \pi/2$ (the case $\theta_0 = 0$ is the trivial case when the motion begins with the ball hole aligned with the spin axis). The first case ($\lambda \to 0$) represents the situation when the hole in the ball is vanishingly small and there is no inertial imbalance to drive the motion. In the second case ($\theta_0 \to \pi/2$) the ball hole is initially aligned normal to the spin axis and the system is in a metastable state of equilibrium which will persist in the absence of an external disturbance. Both cases are in consonance with physical intuition. The criterion in Eq. 2 is further substantiated by comparison with the experimental results reported in Ref. 1.

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